

Since the current concentrations N of CO₂ in the atmosphere is so high, the direct dependence of the surface temperature T on N should be given approximately by

$$T = T_0 + \frac{\Delta T \ln(N/N_0)}{\ln 2} \quad (1)$$

Here T_0 is a reference temperature, say the temperature in 1980 and N_0 is the corresponding atmospheric concentration of CO₂, say 340 ppm at Mauna Loa in 1980. From (1) we see that doubling the CO₂ concentration (letting $N = 2N_0$) will increase T by the “temperature sensitivity” ΔT . The reason for the logarithmic rather than linear dependence on N is the saturation of the CO₂ 15 micron band, that comes from the bending mode of the molecule. Adding more CO₂ slightly broadens the band, as seen from outer space, slightly decreases the amount radiation that is lost to space, and therefore requires a slight surface warming to compensate with more radiation to space through the “infrared window.” One can haggle over a few tenths of a degree, but most theoretical estimates give $\Delta T \leq 1.2$ C for no changes of water vapor and clouds. The argument is over how much “feedback” (changes in water vapor or clouds) will amplify or attenuate the direct sensitivity of 1.2 C.

What is the feedback? One way to estimate the feedback is to look at the observed changes in T and N over past thirty years since satellite measurements of temperature have been available. Satellites actually measure the temperature of the troposphere, not the earth’s surface temperature, but satellite measurements are free of many of the problems that vex attempts to get surface-temperature trends from land-based or sea-based instruments. The satellites actually look at microwave radiation in a strong oxygen band of the atmosphere. Near the band peak the radiation comes from close to the top of the atmosphere, but in the more weakly absorbing sides of the band you can infer the temperature of the lower troposphere. The satellite instruments are calibrated against an on-board black-body source. The satellite data is public and has been scrutinized again and again by those looking for any systematic biases. Indeed some biases have been found and corrected. A slight drift of the satellite orbits, for example, was not at first recognized as a significant factor that could give a cooling bias. As the climategate e-mails have shown, it has been almost impossible to get analogous data for land-based instruments, and even if you can get it, the data has serious systematic problems like urban heat islands and the loss of remote recording stations, which are often from colder locations in Canada or Siberia.

Depending on altitude, latitude on the earth, etc., the tropospheric warming corresponding 1 C of surface warming should be greater, up to 2 C. The reason is that as the surface warms, the surface air can hold more water vapor at the same relative humidity, and therefore more latent heat must be released as the air rises and condenses into clouds of liquid water droplets or ice at higher, cooler altitudes. A representative number for this tropospheric amplification is a factor of 1.4, as discussed in the post below by Steve McIntyre. Below the post I have pasted in a figure from the link to McIntyre site, which discusses the discrepancy between land-based warming trends and satellite-based trends.

If you take the satellite data as least likely to have systematic errors, the warming over the past thirty years has been $dT/dt \approx 0.13$ C/decade.

I have also pasted CO₂ concentration levels measured at Mauna Loa, which have gone from about 340 ppm to 390 ppm from 1980 to 2010 for a concentration growth rate of $dN/dt = 16.6$ ppm/decade.

If we assume that the entire temperature increase of the past 30 years has been due to the CO₂ increase (not very likely) we can find ΔT by differentiating (1) with respect to time t and solving to find (with $N = 365$ ppm)

$$\Delta T = N(\ln 2) \frac{dT}{dt} \left(\frac{dN}{dt} \right)^{-1} = 2\text{C}. \quad (2)$$

The fact that 1980 was the end of the several decades of cooling (of unknown, probably natural origin) and the fact that temperatures seem to have been very stable over the past decade suggest that a good fraction of the warming between 1980 and 2010 has been due to some natural, cyclical cause. If you guess that 1/2 of the warming has been natural (it could well have been more) then the warming potential for doubling CO₂ is only 1 C. The observational data does not permit you to infer anything close to the “most probable” IPCC figure of 3.5 C warming from doubling CO₂.